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COMMENT

Exponent $\nu_\perp$ for 2D directed compact site animals

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Abstract. We obtain the transverse cluster radius exponent $\nu_\perp = 0$ for 2D directed compact animals. We find that the exponent $\nu_\perp$ takes two different values which depend on definition of the animal width.

Recently, a lot of attention has been given to the studies of directed compact lattice animals (Derrida and Nadal 1984, Bhat *et al* 1986, 1987, Privman and Forgacs 1987). The number F_N of distinct N-site animals grows according to

$$F_N \sim \bar{N}^\theta \lambda^N \qquad \text{as } N \to \infty \tag{1}$$

where θ is a critical exponent and λ is a lattice-dependent growth parameter. The generating function for all animals is represented as

$$F(x) = \sum_{N=0}^{\infty} F_N x^N. \tag{2}$$

It exhibits a power law singularity of the form

$$F(x) \sim |x_c - x|^{\theta - 1} \qquad \text{as } x \to x_c. \tag{3}$$

For the fully directed compact site animal model (Bhat *et al* 1986, 1987), relation (1) applies with $\theta = 0$ and $\lambda = 2.661\,85 \pm (5)$. This implies that the singularity of $F(x)$ nearest to the origin is a simple pole at $x = x_c$. The exponents describing length and width of the animals were predicted to be $\nu_\| = 1$ and $\nu_\perp = \frac{1}{2}$ respectively (Bhat *et al* 1987). Recently, Privman and Forgacs (1987) have shown analytically that the animal number exponent $\theta = 0$ for partially directed compact animals.

In this comment, we estimate the transverse cluster radius exponent $\nu_\perp$ for fully directed compact lattice animals in two dimensions. We observe that the exponent $\nu_\perp$ takes two values, which depend on the definition of width.

For two-dimensional directed compact site animals, one can write

$$\text{mass} \sim (\xi_\| \xi_\perp) \tag{4}$$

or

$$N \sim (\xi_\| \xi_\perp)$$

where $\xi_\perp$ and $\xi_\|$ are the transverse and longitudinal cluster radii respectively. From equation (4) one obtains

$$\nu_\| + \nu_\perp = 1. \tag{5}$$

In the case of a directed compact animal on a square lattice, all the sites which are not in the root can be reached from at least one root site by a directed walk of $+X$ or $+Y$ steps between the cluster sites. This implies that $\xi_\parallel$ equals $N/\sqrt{2}$ and hence $\nu_\parallel = 1$ (Bhat *et al* 1987). Thus equation (5) yields $\nu_\perp = 0$. The above result can also be obtained by relating mass to the volume of the cluster (Kinzel 1983, de Queiroz 1984). We write

$$(\text{mass}) \sim (\text{volume})^{d_f/d}$$

or (6)

$$N \sim (\xi_\parallel \xi_\perp^{d-1})^{d_f/d}$$

where d_f is the fractal dimension of the cluster.

From equation (6), we obtain

$$d_f = \frac{d}{[\nu_\parallel + \nu_\perp(d-1)]}. \tag{7}$$

If d_f is less than the spatial dimension d, the animals are highly ramified structures; if $d_f = d$, they are compact (Family and Coniglio 1980). With this argument (i.e. $d_f = d$) and $\nu_\parallel = 1$ for two-dimensional directed compact animals, one obtains from equation (7), $\nu_\perp = 0$.

The exponent $\nu_\perp = 0$ implies that $\xi_\perp$ stays finite as $N \to \infty$ or increases at the most logarithmically. It governs the width of the animal at fixed x. The exponent $\nu_\perp = \frac{1}{2}$ (Bhat *et al* 1987) governs the width of the full animal, i.e. size of the strip to which the animal is confined.

For a strip which is finite in one direction and infinite in the other, the shift in the critical temperature is proportional to $m^{-1/\nu_\perp}$ (Nadal *et al* 1982, Derrida and De Seze 1982). One can thus use finite-size scaling theory, which predicts

$$x_c(m) = x_c(\infty) + A/m^{1/\nu_\perp} \tag{8}$$

where m is the width of the strip and A is a constant. For directed compact animals, we expect the following behaviour:

$$x_c(m) = x_c(\infty) + B\,e^{-mC} \tag{9}$$

where B and C are constants. From equations (8) and (9) we find that

$$1/\nu_\perp = \infty$$

or

$$\nu_\perp = 0.$$

This result for the exponent $\nu_\perp$ satisfies the scaling relation $\theta = \nu_\perp(d-1)$ in $d = 2$ (Family 1982). Future work may lead to the determination of the exponents in higher dimensions and thus a relationship could be established between the exponents θ, $\nu_\parallel$ and $\nu_\perp$.

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